STAT

Test Equipment Mount

DBR 630

17 July 1969

STAT

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Technical Proposal

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17 July 69

CONTENTS



Illustrations



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## 1.0 General

A passive isolator suspension is proposed for the Test Equipment Mount that has a low frequency, viscously damped suspension. The mount will provide isolation and damping for all six degrees of freedom of the isolated mass.. The mount will provide the suspension and floor support. The system will use isolator components already proven in other large optical devices.

## 1.1 Vibration Isolation Design Considerations

The theory of vibration isolation has been well known for many years. All that is needed is a very low frequency suspension with good linear viscous damping in all directions, that isolates and damps all six degrees of freedom of the isolated mass. This has been done successfully for one or two degrees of freedom (i.e., automobile suspensions and aircraft shock struts). The difficulty comes in isolating six modes of vibration input to a mass with six degrees of freedom (i.e., three rotational and three linear).

### 1.1.1 Isolation for Test Equipment

A high quality vibration isolation system requires a six degree of freedom system. As might be expected, in the presence of random six mode vibration input, ideal isolation is achieved when the suspension natural frequencies and damping ratios are equal for all six modes. This avoids any one vibration mode of the suspended mass absorbing an excessive share of the vibratory energy. There seems to be a law in these designs that the weak (i.e., under damped and lowest natural frequency) mode will cause the trouble regardless of the mode and direction of the input. However, in the proposed system true random input does not exist and this must be taken into some account in the isolator design. Also it is evident that the dominant vibration input from the building structure is substantially vertical. With these complexities in mind the spring rates can be planned to attenuate, adequately, the dominant input modes, while at the same time remembering to avoid any weak modes. Avoiding rotational modes is best accomplished by spreading the suspension points at the maximum distance possible from the C.G. of the isolated mass. When an array of isolators are used it is important that the elastic center of the suspension coincides with the C.G. of the mass to avoid "galloping" due to conversion of translational modes into rotational modes by the suspension.

### 1.1.2 Vibration Input Considerations

The vibration input from the building has some dominant low frequencies. These are substantially vertical but are not necessarily in phase from side to side or front to back. The vibration spectrum is complex so that there are typical beat and interference effects which make the input appear to "come and go". Also, there are some high frequency and acoustical type vibrations which are transmitted by the building structure. (Air transmitted acoustical noise is not considered significant.) In the past very low frequency spring suspensions have only been successful if paralleled by a good viscous damper (oleo). However, the metallic springs provided a transmission path for the high frequency noise which can degrade results by affecting optical elements and structure being tested. It is therefore best to break the high frequency transmission path with rubber or elastomers in series with the suspension springs.

### 1.1.3 Low Natural Frequency Suspension vs. Excursion

There are several inherent difficulties with low frequency suspensions that must be overcome. They have very large excursions. Their rest position is affected by small weight differences. The proper approach is to use as low a frequency suspension as the available excursion clearances and use will allow. Of course, low frequency suspensions when not paralleled by viscous dampers have been notorious failures. Fortunately, because of the low vibration inputs, an excellent compromise can be made. A $\pm$ 3/16 inch excursion for a 1.5 Hz suspension can be used with snubbing for all excursions beyond this amount. In this way the inadvertent shock inputs can be contained without damage and the ringing time can be relatively short after shock input. This plan requires a suspension centering adjustment to insure that the full $\pm$ 3/16 inch low frequency excursion is available.

### 1.1.4 Theoretical Attenuation

For the illustrated 1-1/2 Hz suspension with 17-1/2% of critical damping, the theoretical attenuation is shown on the graph, figure 1. Note that a magnification of three occurs at the natural frequency. Because of the nature of the input this is not a difficulty as the vibratory accelerations expected at these low frequencies will be very small. The dominant input frequencies will be attenuated approximately as shown on the graph. Of course, the idealized performance shown is not fully achieved because of resonant effects in the supporting structure and internal to the mount itself.

A series of bumps on the attenuation curve can be expected when the complete equipment is tested. Any such affects can usually be improved by stiffening or damping the offending internal structure. Normally the attenuation of vibrations at frequencies above 1.4 $f_o$ can be made to approach the theoretical curve. Of course all "stiction" and high frequency transmission paths across the isolator suspension must be avoided. Below the natural frequency of the suspension the isolator follows the input motions of the building as shown.

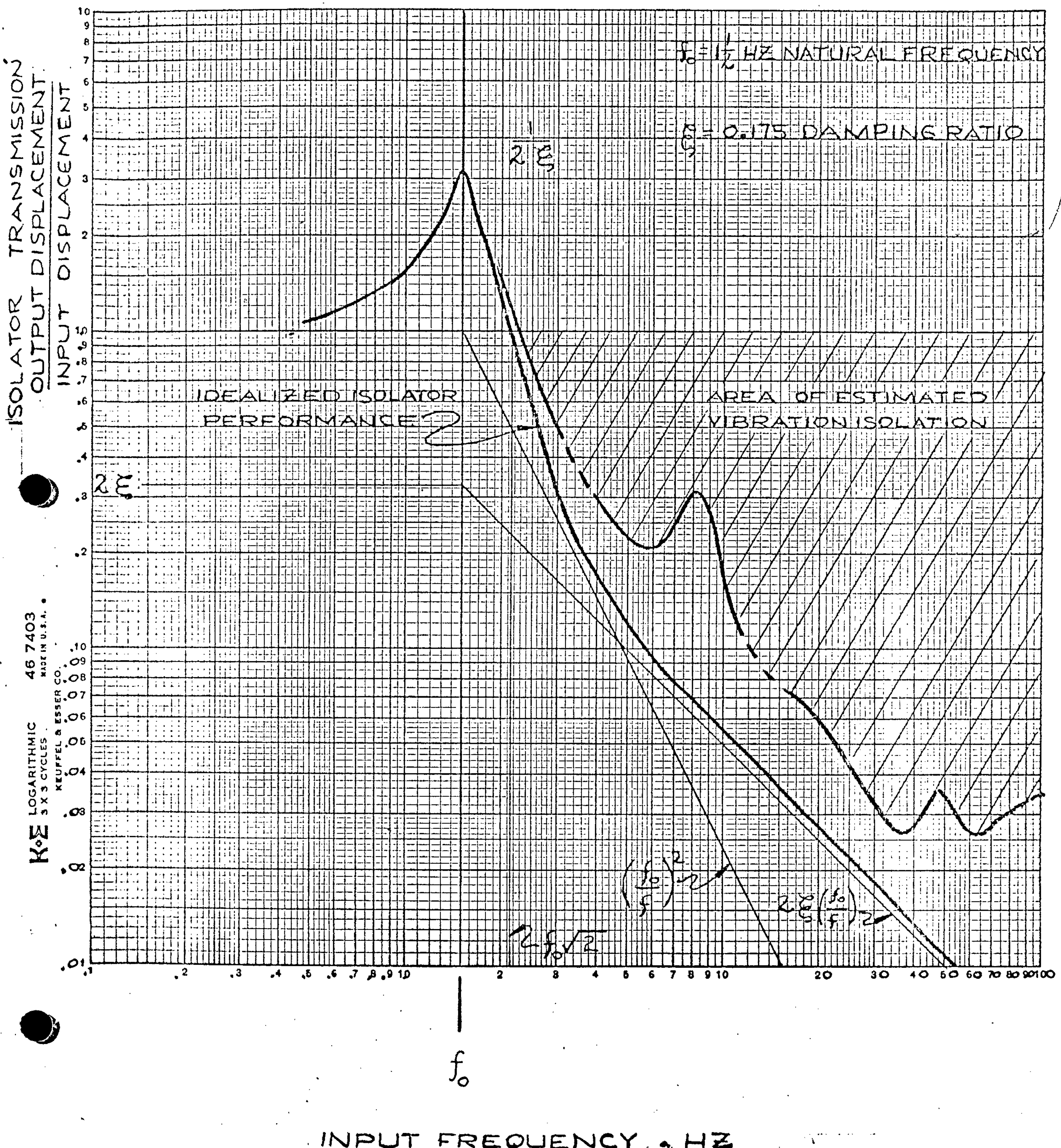
### 1.1.5 Viscous Damper Considerations

Theoretical viscous (linear) damping would be best except for the special conditions of this application. Low damping is desired for the "quiet time" to achieve maximum attenuation (Note that damping relative to the vibrating source is coupling or transmission.) For rough conditions increased damping is desired. By using orifice damping as provided by oleos (shock absorbers) this characteristic is achieved. For quiet times and small excursions the damping force is proportional to fluid velocity. For large excursions during rough conditions, the damping force is proportional to the square of the fluid velocity so that for these large excursions the damping increases significantly. Thus the ringing time is greatly shortened and the rebound from shock inputs is controlled.

### 1.1.6 Isolator Structure Design

It is important that the structure on both sides of the suspension be quite stiff. A good rule of thumb is that it have a natural frequency at lease 10 times that of the suspension for all modes. In this way all significant relative motion occurs across the isolator spring where it can be controlled by the parallel viscous damping. One of the sins of isolator design is to allow storage of spring-mass energy in under damped structure rather than forcing the motion to actuate the oleo damper where the vibratory energy will be dissipated.

# ISOLATOR PERFORMANCE
# VERTICAL AXIS



## 2.0 Isolator Mount Parameters

### 2.1 Isolator

| | | |
|---|---|---|
| 2.1.1 | Low frequency steel spring suspension ($f_o$) (all axes) | 1.2 to 2 Hz |
| 2.1.2 | Excursion (all axes)<br>total without snubbing<br>total including snubbing | <br>$\pm$ 3/16 inch<br>$\pm$ 5/16 inch |
| 2.1.3 | Accelerations at extremes including snubbing | 0.2 G |
| 2.1.4 | Weight of Isolated Mass (estimated) | 5000 lbs. |
| 2.1.5 | Damping | Adjustable. around 17-1/2% of critical |

### 2.2 Environment

| | | |
|---|---|---|
| 2.2.1 | Temperature | Room ambient (see spec.) |
| 2.2.2 | Pressure | Room ambient (see spec.) |
| 2.2.3 | Acceleration | 0.2 G horizontal snubbed<br>1 $\pm$ .2 G vertical snubbed |
| 2.2.4 | Vibration Input | (see specification) |

## 3.0 Detail Discussion

The details of the proposed R-5 Test Equipment Mount, on which the Cost Proposal is based, are discussed in the following Sections:

### 3.1 General Arrangement

The proposed general arrangement is shown in Figure 2. (fold-out in back) Four floor pedestals contain coil springs that support and isolate the Mount. Oleo dampers at the center gravity level of the Mount serve to damp excursions of the Mount at resonance (natural frequency) and allow the attenuation of higher frequency disturbances as shown in Figure 1. The pedestals are set on pads on the floor and are not bolted down. They have adequate floor pads to handle 2/10 "G" overturning force. The excursion of the Mount is snubbed by rubber bumpers internal to the pedestals at the damper attachments. These bumpers provide snubbing in all directions beginning at $\pm 3/16$ inch with maximum excursion at $\pm 5/16$ inch. Leveling adjustments are provided at each pedestal so that the Mount can be adjusted level and to the center of the unsnubbed traverse.

### 3.2 Mount Suspension

We propose to use steel coil springs to provide the low spring rate necessary for a nominal 1-1/2 Hz suspension. By supporting the springs on neoprene pads, the transmission of high frequency disturbance through the steel of the coil spring is blocked. By this means excellent isolation is provided. To further block high frequency disturbance the pedestals supporting the spring mounts on the floor are also placed on attenuating pads of standard commercial type. The coil springs are designed to have a transverse spring rate, when loaded, approximately the same as their vertical spring rate so that the suspension natural frequencies, both vertical and horizontal, will very nearly match (see Section 2.1.1). The one "G" deflection of a Spring-Mass System versus Natural Frequency (resonant frequency) is given in Figure 4. Note that a 1-1/2 Hz natural frequency requires a spring rate measured by a four-inch, one "G" deflection.

#### 3.2.1 Rotational Mode - Longitudinal Axis

The weakest oscillatory mode of the proposed suspension is rotational about the longitudinal axis of the test mount. In spite of placing the vertical support springs in the pedestal out at 27 inches from the center line (and encroaching on some additional floor space (see Figure 5), the rotational natural frequency about the longitudinal axis is only 0.8 Hz which is too low as it will be the weakest mode. Therefore, additional longitudinal torsional restraint is planned by means of

# SPRING-MASS SYSTEM

## 1.0-g DEFLECTION vs NATURAL FREQUENCY

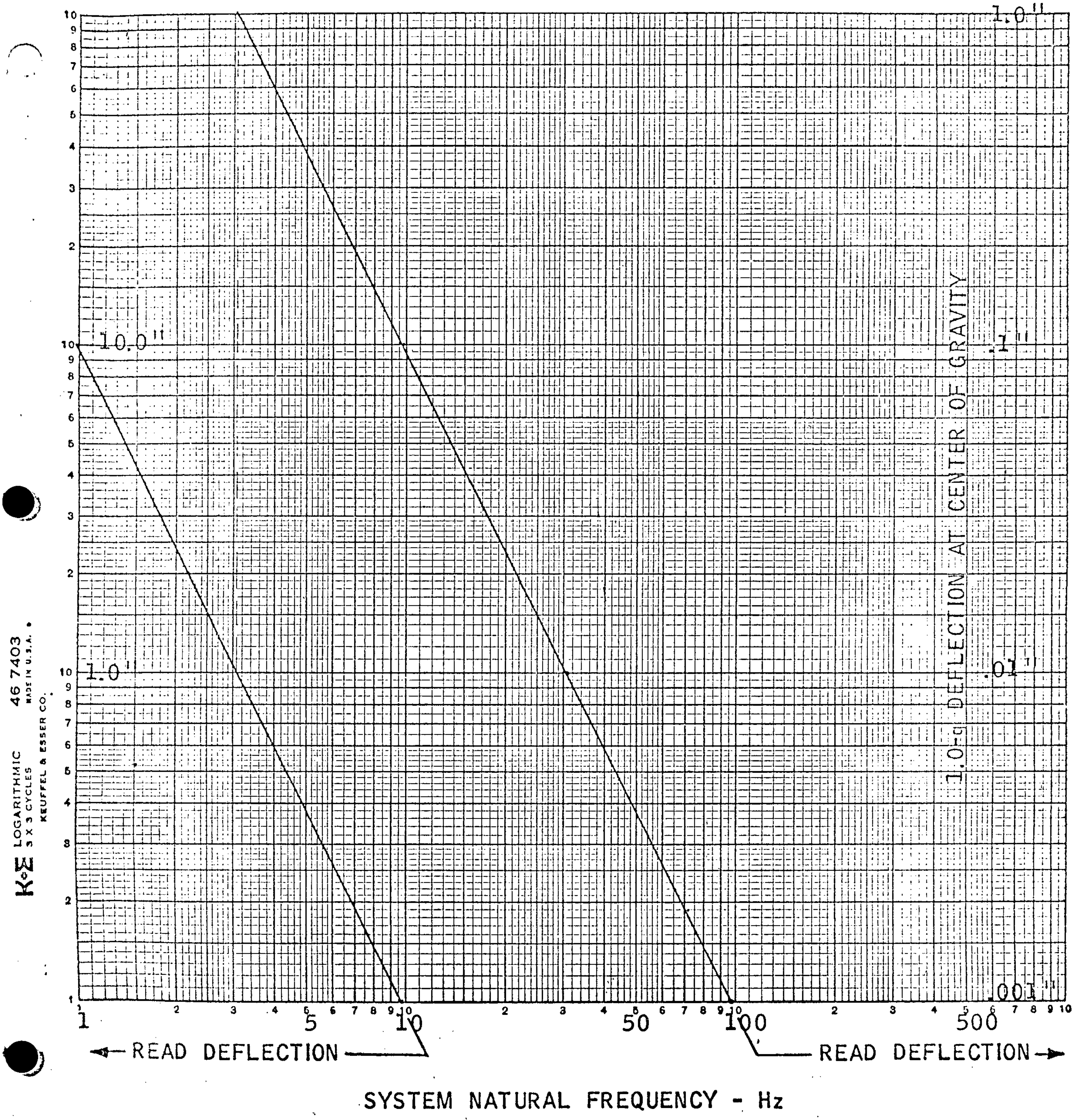



FIGURE 4

3.2.1 Rotational Mode - Longitudinal Axis (Contd.)

torsion rods mounted internally to the test mount structure. These are analgous to roll stabilizers in automobile suspensions. A 1-1/2 to 2 Hz roll natural frequency is planned.

## 3.3 Dampers (DMI Part No. 20380

DMI adjustable orifice oleo dampers, illustrated in Figure 3, are proposed to provide linear (viscous) damping paralleling the coil springs. These dampers will be used to provide vertical and horizontal damping. One damper at each of the eight support springs will be installed vertically to control vertical motion. One damper at each of the four center support springs will be installed horizontally to control transverse motion of the mount. One damper at each of the four end support springs will be installed horizontally to control longitudinal motion of the isolated mount. Thus a total of sixteen oleos are used. These dampers will be adjusted to provide approximately 17-1/2 per cent of critical damping as indicated in Figure 1. The individual damping orifice is adjustable to allow optimizing the damping as necessary for both horizontal and vertical motions.

### 3.3.1 Damper Details (See Figure 3)

The dampers are filled with silicone fluid of high viscosity. The fluid is contained between two Bellofram pistons which pump the oil through the central orifice plate when the push rod is actuated. A thermal relief and pressurizing piston is provided to allow for expansion of the fluid through a capillary orifice. This capillary is small so that the relief piston does not enter into the dynamic system. All the fluid is contained by the Belloframs. There are no sliding seals so that the dampers do not leak fluid and have no sliding friction. The push rod is guided by a linear ball bearing in the body and a Rulon sleeve in the orifice plate, thus minimizing "stiction" all possible. The "ball end" attachments on the body and the push rod allows three degrees of freedom motion of the ends while still providing damping for the motion controlled by the particular unit. The unit provides a very smooth damping force proportional to the velocity of motion for small excursions and approximately proportional to the square of the velocity for large excursions. This last occurs because the orifices are "short pipes" rather than sharp edged orifices that would produce velocity squared forces. Thus for small motions during normal use of the test mount, the ideal linear or "velocity proportional" damping force is provided. For large inputs or shock, the dampers stiffen-up and provide forces nearly proportional to velocity squared.

# OLEO DAMPER
# DMI P/N 20380

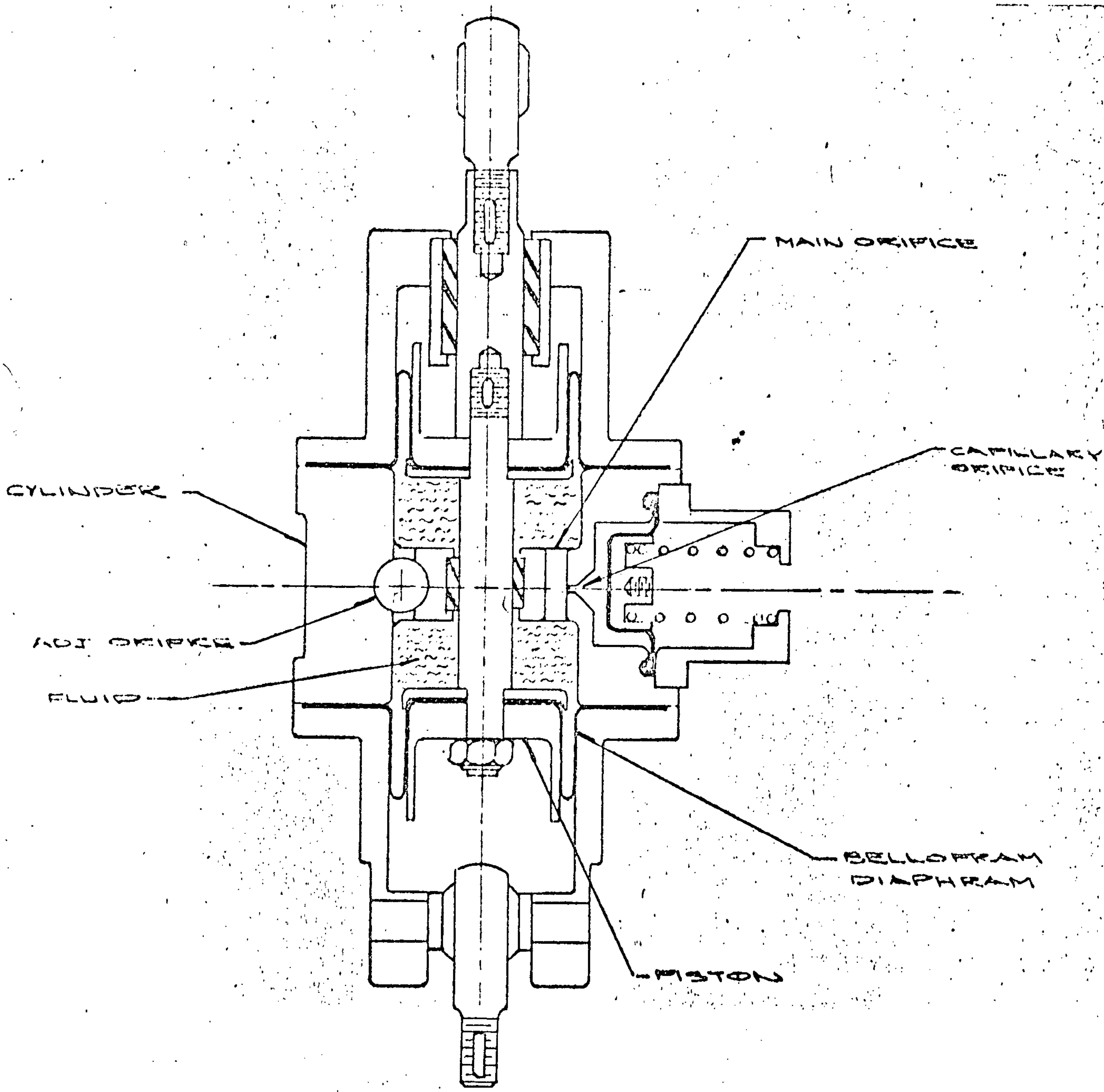



Figure 3

## 3.4 Pedestals -(See Figure 6)

The pedestals are proposed to be aluminum alloy weldments of square tube. Four pedestals, each with two spring supports, are planned as shown in Figure 2. They will be designed to spread the loads so that the floor loading will not exceed 250 lbs. per square foot. In addition, they are designed to have wide enough floor pads to withstand an 0.2 "G" horizontal load without overturning. This is a generally accepted safe loading concept including all safety factors. The surfaces will be smooth and corners rounded. They are planned to be set on isolation pads on the floor. Metal straps between the pedestals at floor level will serve to locate and hold them in position. They will not be bolted to the floor.

## 3.5 Floor Plan (See Figure 5)

It is proposed that the space allocated in the specification for the test mount be increased slightly to allow the suspension to be external to the mount structure. This is necessary to avoid discontinuities in the mount structure that would seriously impair its stiffness if the suspension was installed inside the mount frame. Therefore, the pedestals for the suspension springs are widened to 60 inches rather than the 48 inches shown in the specification. Of course, the extra width on the side away from the wall only occurs at the pedestals, leaving the majority of the mount unobstructed (see Figure 5).

## 3.6 Test Mount Structure

The test mount structure is proposed to be an aluminum alloy bolted and epoxied assembly. A box shape is proposed to provide the maximum stiffness the space envelope will allow. Even so, the desired rigidity of the structure (i.e., 90 Hz) cannot be met within the space envelope. The stiffness that can be provided within the space envelope is estimated and is considered adequate for the described use. However, in view of the low natural frequency of the total structure and to avoid "ringing," it is desirable that a fabrication technique be used which will inherently provide internal damping.

### 3.6.1 Structural Stiffness

The space envelope determines the maximum structural stiffness possible in the test mount. The concept under consideration is the fundamental natural frequency of the structure in the vertical and horizontal simply supported beam modes. In addition, the rotational fundamental natural frequency of the structure is also a consideration. There is no shape within the space envelope which will provide a

ISOLATOR STRUCTURE LAYOUT

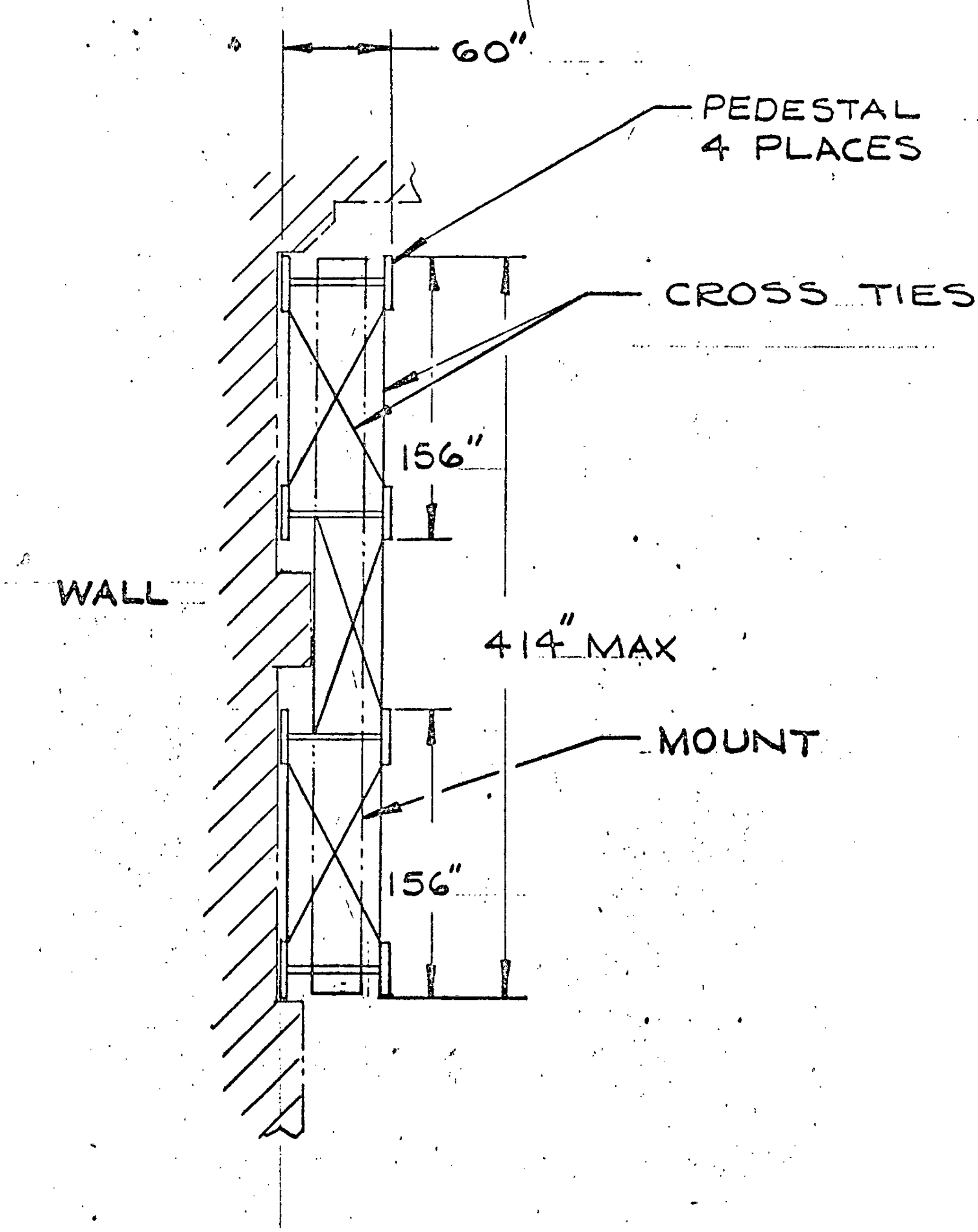



FIGURE 5

### 3.6.1 Structural Stiffness (Contd.)

maximum natural frequency in that the maximum occurs when the beam caps reach zero and the weight becomes zero. In other words, the only design concepts that can be used are those of practical nature involving local deflections, concentrated loading, equipment alignment, and practical fabrication problems. Because the transverse and rotational fundamental natural frequencies must be considered, as well as the vertical, the design is then largely dictated. We have chosen beam caps one inch thick and box sides (beam webs) of one-quarter inch thick. Using these dimensions, following are the estimated fundamental natural frequencies of the free body:

| | |
|---|---|
| Vertical | 30 Hz |
| Transverse | 18 Hz |
| Torsion | 47 Hz |

As can be seen, these are considerably under the specification statement. However, analysis shows that the test mount will be adequate even at these low natural frequencies. A rule of thumb in these designs is that if the structural natural frequencies are at least ten times higher than the isolator suspension natural frequency, the devices will be satisfactory.

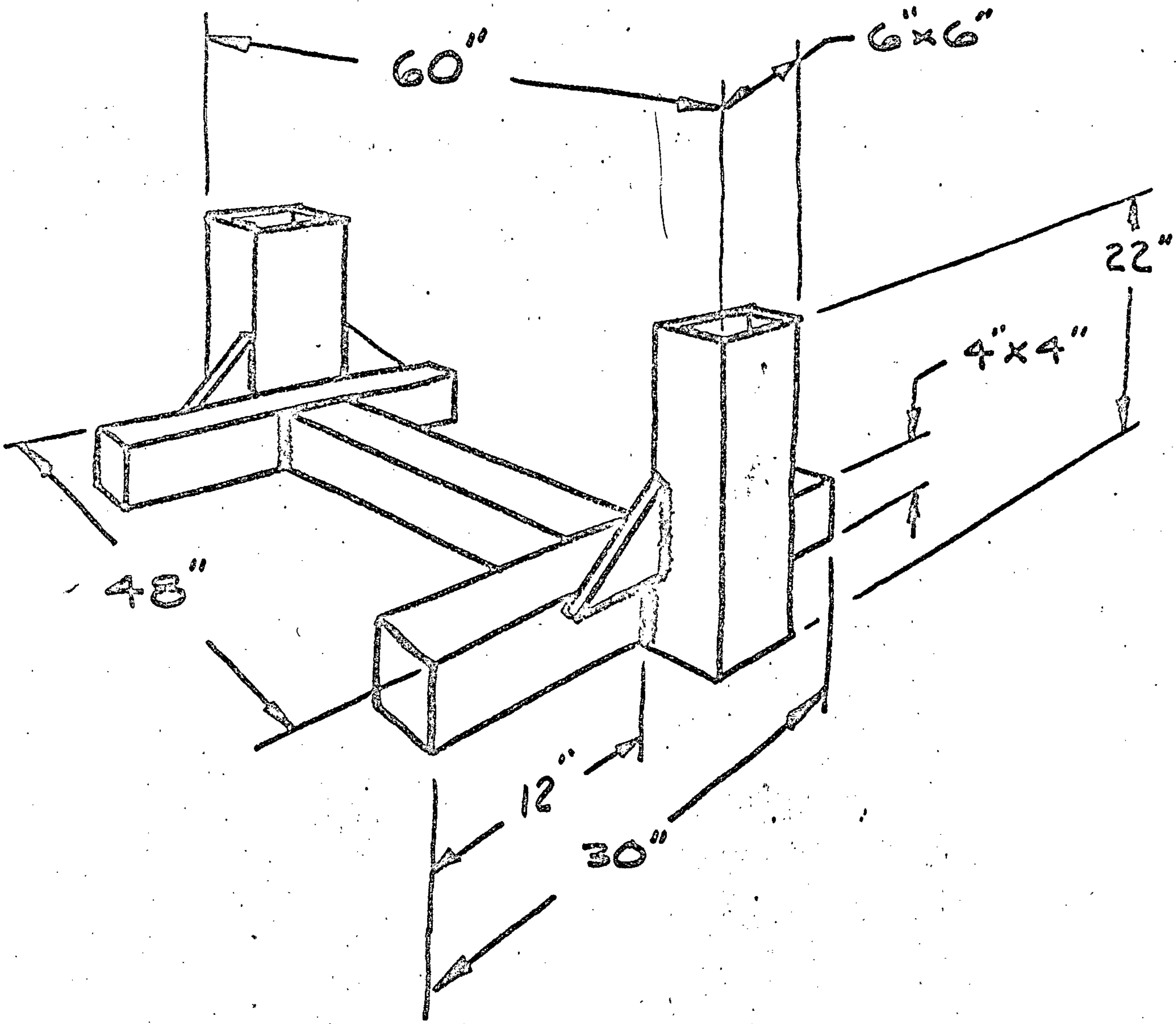
### 3.6.2 Mount Beam Deflections

Because of the lower-than-requested natural frequency, investigation of the angular deflections of the test mount surfaces under vibration excursion is made. Using the estimated transmissability (Figure 1) and the specification vertical inputs, it appears that the maximum expected is an eleven microinch, peak to peak, excursion of the beam at the center relative to the ends. On this basis, the angular excursion of the beam ends, in the fundamental mode, is .05 micro radians, peak to peak. This represents an optical axis motion at the end of the beam of approximately 1/2000 millimeters. Further, the change in angular slope of the beam surfaces when the 400 lb. concentrated loads are shifted from the center to the ends is estimated at less than five micro radians for the optical axis at the center.

### 3.7 Material for Mount Beam

As the design is primarily a stiffness design rather than strength, either steel or aluminum alloy can be used. The elastic modulus versus weight ratio of either

# TYPICAL PEDESTAL



3.7 Material for Mount Beam (Contd.)

metal is nearly the same and is the most favorable of any commonly used material. Only beryllium or titanium have more favorable ratios and both of these have cost and fabrication disadvantages.

3.8 Fabrication Techniques (See Figure 7)

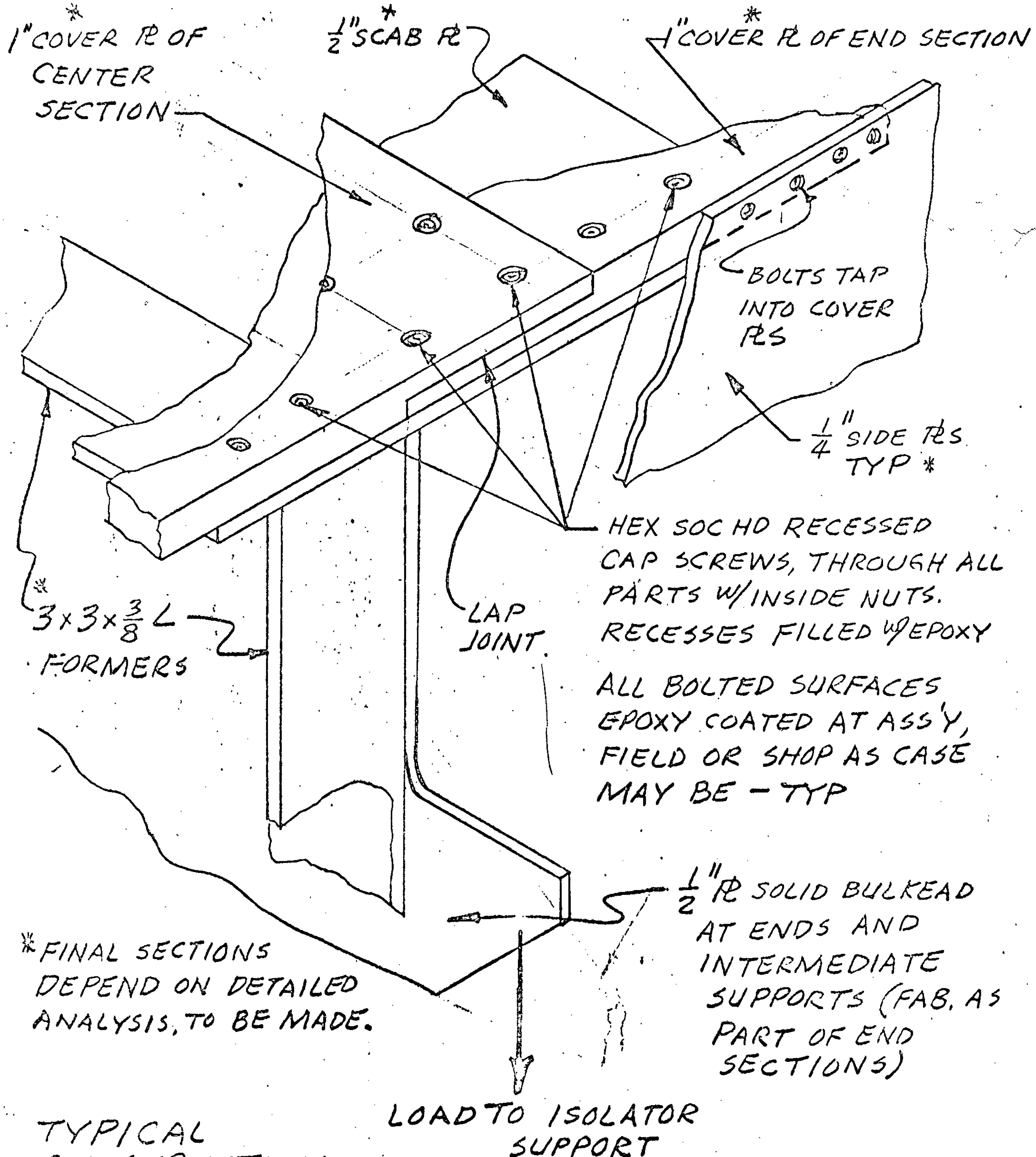
To achieve an inherently damped structure, a bolted and epoxied fabrication method is chosen. This method forbox structures allows the use of as-rolled plate stock material, it merely being necessary to saw cut and machine the edges at the bolted corners. By using epoxy as a seating compound as well as to lock and key the fabrication bolts, a continuous transfer of elastic strain is achieved, providing a stable but damped structure. Locking the permanent fabrication bolts with epoxy allows the use of tapped holes in aluminum as the threads are not required to survive removal or generate much tensile strength. Essentially all strain at the corners is transferred by the epoxy in shear. As the mount structure is to be fabricated in three pieces, provision must be made to provide good strain transfer at the assembly joints. It therefore is planned that the final assembly of these joints at Kodak will also be completed using epoxy as a seating compound and to lock all permanent bolts. Figure 7 shows a detail at one of these assembly joints.

3.8.1 Stabilized Structure

As the end use requires that the test mount have excellent dimensional stability, it is planned to give all the structural rolled plates a stabilizing heat and cold cycle treatment after machining. This, together with the use of the epoxy material as a seating compound in the joints, will provide excellent dimensional stability.

3.9 Test Technique

Simple impulse-type tests are planned to evaluate the isolation system at  STAT Direct momentary impulses will be given to the suspended structure in vertical and horizontal modes. The resultant motion die-out will be recorded using geophones. From these recordings natural frequencies and damping ratios will be derived. An approximate evaluation of the transmission through the isolators will also be made by comparing geophone recordings of motion on the floor adjacent to the pedestals and on the Mount. This last should be a good representation of actual use as considerable seismic noise is generated in the  plant by the adjacent highway. STAT




TYPICAL CONSTRUCTION DETAILS

FIGURE 7

## 4.0 Comments on Specification

To assist in the detail understanding of our proposal, we have included pertinent comments on the Specification by paragraph.

| Spec. Paragraphs | Comments |
|---|---|
| 1. through 1.3 | No comment |
| 2. | On the basis that documents are applicable only to the extent of specific reference elsewhere in the Specification, no comment is required here. |
| 3. through 3.1.1 | No comment |
| 3.1.2 | The isolation system we propose is a passive steel spring - elastomer - oleo damped suspension not requiring active air pressure or control. |
| 3.2 through 3.2.2 | No electric power is used in our proposed passive isolation system. |
| 3.3 through 3.3.1 | We propose that locally at the four points of support the suspension mounts extend beyond the envelope approximately eight inches. |
| 3.3.1.1 | Our proposed lengths and weights are given in Paragraph 2.1.4 of our proposal and will be within the limitations. |
| 3.3.2 | The weight will be less than 20,000 lbs. |
| 3.3.3 | The working height will be nominally 42 inches. Leveling adjustments are provided which may exceed these limits to the extent the floor level requires it. |
| 3.3.4 | The proposed design accommodates loads. Slight changes in level may result from shifting the concentrated loads. |

4.0 Comments on Specification (Contd.)

| Spec. Paragraphs | Comments |
|---|---|
| 3.3.5 | The nominal springs-mass suspension natural frequency will be nominally 1.2 cps vertically but may vary slightly up to 1.7 cps and the translational (horizontal plane) natural frequencies may vary from 1.2 cps up to 2 cps. These variations result from the effect of the sequential spring rates of the support structure on both sides of the isolator springs. These are not determinate by reasonable analytic means and the effective spring rate is only precisely known after assembly and test of the complete device. |
| 3.3.6 through 3.3.6.2 | Pads and angles will be provided and machined as specified. |
| 3.3.6.3 | The natural frequency of the mount structure (transverse simply supported beam mode) will be 30 Hz vertical and 18 Hz horizontal as discussed in Section 3.6.1. It is not possible with ordinary materials (steel or aluminum) to meet the specified 90 Hz within the 42" x 30" dimensional envelope. In fact, no known material or fabrication method will meet the specified stiffness requirement. |
| 3.3.7 through 3.3.7.2 | The isolator system is proposed to be a spring-mass system paralleled by oleo dampers as described in Sections 3.2 through 3.3.1. This system will meet or better the isolator performance of the "air-spring" system. |
| 3.3.7.3 | Rubber-faced snubbers are provided to limit the isolated mass excursion under high motion or force inputs (i.e., earthquakes or inadvertent impacts). |

## 4.0 Comments on Specification (Contd.)

| Spec. Paragraphs | Comments |
|---|---|
| 3.3.7.4 through 3.3.7.7 | No air supply is used. The oleo isolators will be sealed and do not leak oil. Belloframs avoid all sliding seals. |
| 3.3.7.8 | The supports for the isolators are designed to be stable without being bolted to the floor. |
| 3.3.7.9 | The spring-oleo damped isolators are spread apart to provide roll stability. |
| 3.3.8 | No air supply is used. |
| 3.3.9 | Welding standards as specified will apply; however structural welds will be limited to the isolator supports only. |
| 3.3.10 through 3.3.11.1 | To be as specified. |
| 3.3.12 | No quick release devices are anticipated in the proposed design. |
| 3.3.13 through 3.3.16 | To be as specified. |
| 3.3.17 | Sections will be provided with alignment dowels. |
| 3.4 through 3.4.2 | To be as specified. |
| 3.4.3 | Recommend and propose shipping by door-to-door furniture van which does not require crating. Skids will be provided. |
| 3.5 through 3.5.3 | To be as specified or as approved by EKCo during design. |

4.0 Comments on Specification (Contd.)

| Spec. Paragraphs | Comments |
|---|---|
| 3.5.4 | The design proposed does not have a hazardous failure mode. |
| 3.5.5 | [redacted] standard Q.C. practices will be used and STAT are those used on previous activity. |
| 3.5.6 | The proposed test equipment mount deviates from this Specification as delineated in this proposal. |
| 3.5.7 through 3.5.9.3 | To be as specified. |
| 3.5.10 | The structure will be primarily aluminum alloy iridited and painted with specified epoxy paint. It is assumed that this finish will satisfactorily meet the washing requirement. |
| 3.5.11 through 3.5.12 | To be as specified. |
| 3.6.1 | (a)(b) Drawings will be similar to Stage 4 Drawings with which we are familiar from past work. We are not acquainted with Form 3 Drawings. |
| | (c) The oleo dampers are [redacted] proprietary STAT parts, Part No. 20380, and will be treated as commercial parts. |
| | (d) No modifications to commercial devices are anticipated. |
| | (e) thru (l) To be as specified. |

4.0 Comments on Specification (Contd.)

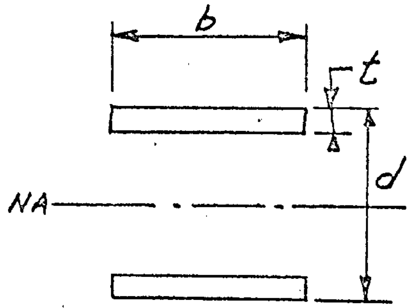
| Spec. Paragraphs | Comments |
|---|---|
| 3.6.2 | This Section of the Specification is construed to apply where applicable to the passive spring-oleo damped suspension. In particular, Items b.2 and b.4 do not apply. Item b.6 is construed to mean only typed on bond paper and with 8" x 10" glossy photo prints. |
| 3.6.3 | The test procedures proposed do not contemplate a full scale vibration test. Simple simulation and vibration tests will be used. (See Section 3.9) Procedures will be as specified. |
| 3.6.4 | [redacted] standard and previously approved Quality Control Plan will be used. STAT |
| 3.6.5 through 3.6.6 | No comment |
| 3.7 | The schedule contemplates approval for fabrication on a piece-meal basis beginning immediately after the Preliminary Design Review Meeting. |
| 3.8 | No comment |
| 4. through 4.1 | See previous comments. |
| 4.2 through 4.2.1 | [redacted] assumes Section 3.6.3 will be modified to conform to their proposed natural frequencies in view of the unattainability of the specified frequency of 90 Hz within the space envelope. STAT |
| 4.3 and 4.4 | Satisfactory as stated to [redacted] STAT |
| 5. through 5.1 | [redacted] presumes these Sections can be construed to conform to a direct door-to-door furniture van shipment. The units will be mounted on skids. STAT |
| 5.2 and 5.3 | This Section is construed to mean all small loose parts. |

Calculate Data for Freq vs Flange Thickness

Fundamental Bm. Mode Deflection $y_{max} = \frac{5 w \ell^4}{384 EI}$ simple/uniform

Holding → $\ell$, E constant, $y_{max} = \frac{K_1 w}{I}$ $K_1 = \frac{5 \cdot \ell^4}{384 E}$

Res. Frequency $= \frac{3.12}{\sqrt{y_{max}}}$ Hz $= K_2 \sqrt{\frac{I}{w}}$ $\frac{I}{w} = \frac{F^2}{K_2^2}$ $K_2 = \frac{3.12}{\sqrt{K_1}}$




$I_{flanges\ only} = \frac{b}{12}\left[d^3 - (d-2t)^3\right]$

or $I = K_3(4t^3 - 6dt^2 + 3d^2t)$

where $K_3 = \frac{b}{24}$

Now $w = f(d,t)$. assuming b constant

When $t=0, w=0$ When $t = \frac{d}{2}, w = max\ (12bd\rho)$
where $\rho$ = density in pci units.

For any $t$, $w = 2 \cdot t \cdot b \cdot 12 \cdot \rho = 24bt\rho$

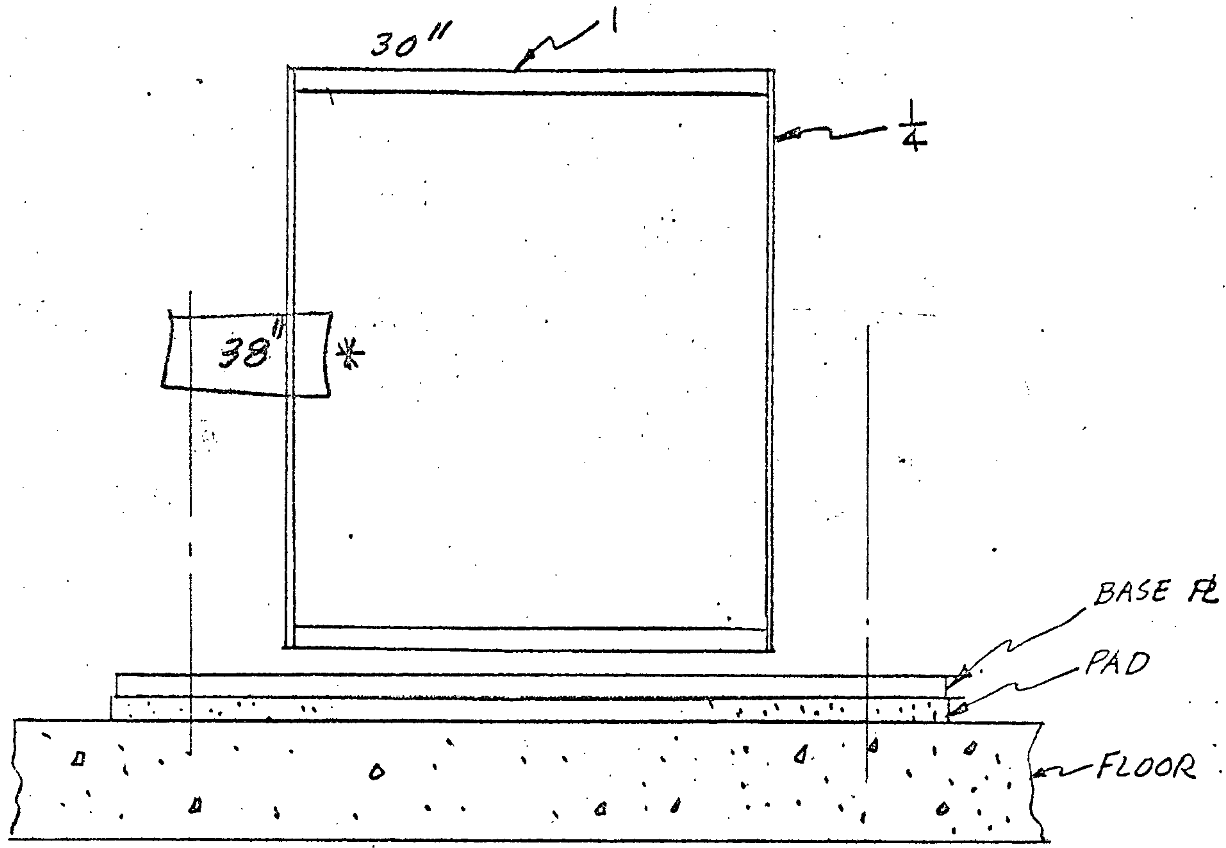
Then $\frac{I}{w} = \frac{K_3}{24bt\rho}(4t^3 - 6dt^2 + 3d^2t) = \frac{1}{(24)^2\rho}(4t^2 - 6dt + 3d^2)$

For $\frac{I}{w}$ min, $f'(t) = 0$, $8t - 6d = 0$ $t = \frac{3}{4}d$ (imaginary root)

∴ There is no real max $\frac{I}{w}$.

For <u>own</u> weight. Min $t$ gives highest frequency.

TRIAL SECTION & ANALYSIS




* NOTE ANALYSIS THEN REMADE FOR 40.5" EFFECTIVE DEPTH.

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## TORSIONAL RIGIDITY OF TRIAL SECTION

$\theta = \frac{\tau L}{KG}$ (STIFFNESS STATEMENT) where $\tau$ = torque

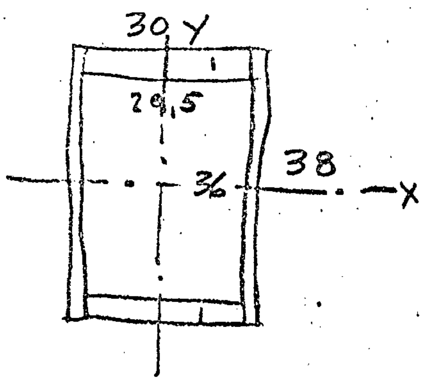
Frequency, $F = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{G J_p}{I_p L} g}$

$J_p$ = polar area moment of inertia

$I_p$ = mass polar moment of inertia.

$g = 32.2$ ft/sec² $L$ = ft.

By definition $J_p = I_x + I_y$




$$I_x = \frac{30(38)^3}{12} - \frac{29.5(36)^3}{12}$$

$$I_y = \frac{38(30)^3}{12} - \frac{36(29.5)^3}{12}$$

$I_x$ = 22,500 in⁴

$I_y$ = 8,650 in⁴

$J_p = I_x + I_y$ = 31,150 in⁴

$I_p = \frac{M}{A} \cdot J_p$

$A = (30)(38) - (29.5)(36) = 78 \text{ in}^2$

$L$ = 406 in.

$M = 78 \text{ in}^2 \times 406 \text{ in} \times 0.1 \text{ lb/in}^3 = 3{,}170 \text{ lb.}$

$G = 3.75 \times 10^6 \frac{\text{lb}}{\text{in}^2}$

$I_p = \frac{3{,}170 \text{ lb}}{78 \text{ in}^2} \cdot 31{,}150 \text{ in}^4 = 1.265 \times 10^6 \text{ lb-in}^2$

$$\therefore F_T = \frac{1}{2\pi}\sqrt{\frac{3.75\times10^6 \times 3.115\times10^4 \times 386}{1.265\times10^6 \times 406}} \quad \left(\frac{\text{lb}}{\text{in}^2}\cdot\frac{\text{in}^4}{\text{lb-in}^2}\cdot\frac{\text{in}}{\text{sec}^2}\cdot\frac{1}{\text{in}} = \frac{1}{\text{sec}^2}\right)$$

$$F_T = \frac{1}{2\pi}\sqrt{8.95\times10^5} = \frac{1}{2\pi}\sqrt{.9\times10^4} = \frac{300}{2\pi} = \underline{\underline{47 \text{ Hz}}} \ *$$

* see retrial for 40.5" effective depth, but $F_T$ does not change significantly.

VERTICAL FLEXURAL MODE

$$y'_{max} = \frac{5Wl^3}{384EI_x} \qquad W = 3{,}170 + (30\,\text{lb})(35') + 600 = 5{,}000\text{ lb.}$$

$$y'_{max} = \frac{(5)(5{,}000)(406)^3}{384(10.5)(10^6)(22{,}500)} = .0185$$

$$y''_{max} = \frac{Pl^3}{48EI} = \frac{400\,(406)^3}{48 \times 10.5 \times 10^6 \times 22.5 \times 10^3} = .002$$

$$F = \frac{3.12}{\sqrt{y'_{max}}} = \frac{3.12}{\sqrt{1.8 \times 10^{-2}}} = \frac{3.12}{.135} = \underline{\underline{23 \text{ CPS}}} \;*$$

LATERAL FLEXURAL MODE

$$y'_{max} = \frac{I_x}{I_y} \cdot .0185 = \frac{22{,}500}{8{,}650} \times .0185 = .048$$

$$F = \frac{3.12}{\sqrt{.048}} = \frac{3.12}{.22} = \underline{\underline{14 \text{ CPS}}} \;*$$

* See retrial for 40.5" effective depth.

RE-ANALYSIS FOR 30" WIDE x 40.5" EFFECTIVE DEPTH

$$I_x = \frac{30(40.5)^3 - 29.5(38.5)^3}{12} = 25{,}000 \text{ in}^4$$

$$I_y = \frac{40.5(30)^3 - 38.5(29.5)^3}{12} = 9{,}150 \text{ in}^4$$

$$J_\rho = 34{,}150$$

$$A = 30(40.5) - 29.5(38.5) = 80 \text{ in}^2 \qquad M = 80 \times 406 \times .1 = 3250 \text{ lb.}$$

TORSION

$$I_\rho = \frac{3{,}250}{80} \cdot 34{,}150 = 1.39 \times 10^6$$

$$F = \frac{1}{2\pi}\sqrt{\frac{3.75 \times 10^6 \times 3.415 \times 10^4 \times 386}{1.39 \times 10^6 \times 406}} = \frac{1}{2\pi}\sqrt{8.7 \times 10^4} = \underline{\underline{47 \text{ Hz}}}$$

VERT. FLEXURE

$$F \text{ higer by } \sqrt{\frac{25{,}000}{22{,}500}} = 23 \text{ Hz} \times 1.05 = \underline{\underline{24 \text{ Hz}}} \times 1.25 =$$

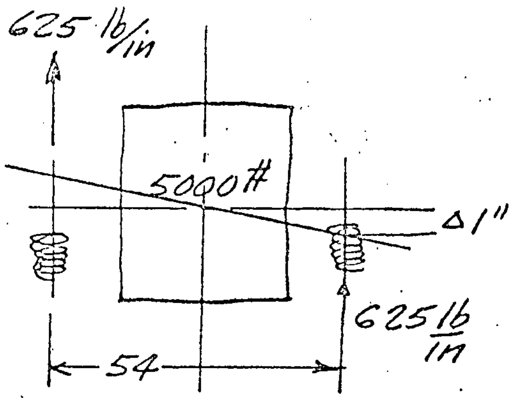
$\underline{30 \text{ Hz}}$ for simple BEAM MODE

LATERAL FLEXURE

$$F = 14 \text{ Hz} \times \sqrt{\frac{9{,}150}{8{,}650}} = 14 \times 1.04 = \underline{\underline{14\tfrac{1}{2} \text{ Hz}}}$$

$\times 1.25 = \underline{18 \text{ Hz}}$ for simple BEAM MODE

# ROTATIONAL MODE FREQUENCY OF THE SUSPENDED MASS




Vertical $\Sigma k = \frac{5000\#}{4 \text{ in}} = 1250 \text{ lb/in}$

For $\Delta 1''$, $\phi = \frac{1}{27} = .037$ radian

Torque $T' = 625\# \times 54'' = 3{,}380$ in-lb.

$$\frac{T'}{\phi} = \tau = \frac{3380 \text{ in-lb}}{.037 \text{ rad}} = 9.1 \times 10^4 \frac{\text{in-lb}}{\text{rad.}}$$

Rotational Period, $T = 2\pi\sqrt{\frac{I_p}{\tau g}} = 2\pi\sqrt{\frac{1.39 \times 10^6}{9.1 \times 10^4 \times 386}}$

$I_p = 1.39 \times 10^6 \text{ in}^2\text{-lb.}$

$$= 2\pi\sqrt{3.96 \times 10^{-2}} = \underline{1.25 \text{ sec.}}$$

$\text{Freq} = n = \frac{1}{T} = \underline{0.8 \text{ Hz}}$ Too Low

We would like at least 2 Hz

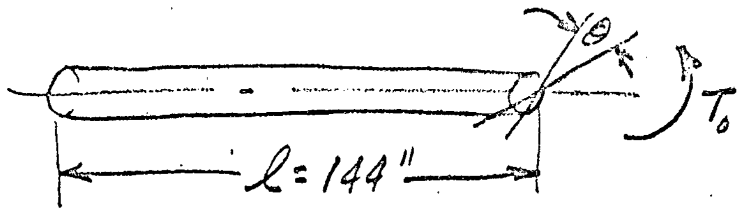
$n = \frac{1}{2\pi}\sqrt{\frac{\tau g}{I_p}}$, $\tau_r = \frac{I_p^2 n^2 (4\pi^2)}{g}$ Req. $\tau_r$

$$\tau_r = \frac{1.39 \times 10^6 \text{ in}^2\text{-lb} \times 4 \frac{1}{\text{sec}^2} \times 4\pi^2}{386 \text{ in/sec}^2} = 57.5 \times 10^4 \frac{\text{in-lb}}{\text{rad}}$$

Springs furnish $- 9.1 \times 10^4$

A torsion shaft must provide $48.4 \times 10^4 \frac{\text{in-lb}}{\text{rad}}$

## Design Torsion Shaft Flexures  Steel




$$\theta = \frac{T_0 \ell}{kG}$$

$$\text{or } k = \frac{T_0}{\theta} \cdot \frac{\ell}{G}$$

Each flexure to provide a $\frac{T_0}{\theta}$ of $24 \times 10^4 \ \frac{\text{in-lb}}{\text{rad.}}$

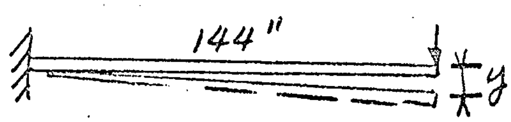
$$K_{req} = 24 \times 10^4 \ \frac{\text{in-lb}}{\text{rad}} \times \frac{144 \text{ in}}{11 \times 10^6 \ \frac{\text{lb}}{\text{in}^2}} = \underline{3.14 \text{ in}^4}$$

Solid Shaft $k = \frac{\pi r^4}{2}$  $r^4 = \frac{2K}{\pi} = \frac{2 \cdot 3.14}{\pi} = 2 \text{ in}^4$

$r_{req} = 1.19$  $d = 2.38 = \underline{2\frac{3}{8}'' \text{dia}}$

Alloy Steel

## Vertical Spring Rate




$$y = \frac{P\ell^3}{3EI_x} \qquad \frac{P}{y} = \frac{3EI_x}{\ell^3}$$

$$I_x = .786 r^4 = .786 \times 2 = 1.57 \text{ in}^4$$

$$\text{Spring rate, each} = \frac{k}{2} = \frac{P}{y} = \frac{3 \times 29 \times 10^6 \text{ lb}}{\text{in}^2} \times \frac{1.57 \text{ in}^4}{1.44^3 \times 10^6 \text{ in}^3}$$

$$\frac{k}{2} = 45 \text{ lb/in}$$

Total Vertical Spring Rate Added $= k = 90 \ \frac{\text{lb}}{\text{in.}}$

This adds to present $1250 \ \frac{\text{lb.}}{\text{in.}}$ — not significant.

$$\text{Wt rods} = 288 \text{ in} \times 4.45 \text{ in}^2 \times .284 \ \frac{\text{lb}}{\text{in}^3} = \underline{364 \text{ lbs}}$$

Cont. Torsional Shaft Flexures

Simple span deflection of shafts (own weight)

$$y_{max} = \frac{5 w l^3}{384 EI} = \frac{5 \times 182\ lb \times \overline{1.44}^3 \times 10^6\ in^3}{384 \times 29 \times 10^6 \frac{lb}{in^2} \times 1.57\ in^4} = \underline{\underline{.156\ in}}$$

$$n_r = \frac{3.14}{\sqrt{.156}} = \frac{3.14}{.384} = \underline{\underline{8.2\ Hz}}$$ Res freq of shaft

If shaft supported but not clamped at midspan.

$$y_{\frac{1}{4}span} = \frac{.156}{8} = .02\ in. \quad n_r = \frac{3.14}{\sqrt{.02}} = \underline{\underline{22\ Hz}}$$

If shaft supported @ $\frac{1}{3}$ span points

$$n_r = \underline{\underline{42.5\ Hz}}$$

Additional Data Relative to Technical Proposal - Appendix

Test Equipment Mount

In the preliminary engineering calculations for subject proposal, we estimated the vertical flexural, lateral flexural and torsional mode natural frequency fundamental vibrations of the isolated structure/mass. They were:

Vertical, 30 Hz / Lateral, 18 Hz and Torsional, 47 Hz , based

on a structure nominal cross-section plus appurtenances and superimposed loads totaling a supported mass of approximately 5,000 lbs. For that section, $I_x$ = 22,500 in$^4$, $I_y$ = 8,650 in$^4$, $J_p$ = 31,150 in$^4$ ($I_x + I_y$) and $I_p = 1.265 \times 10^6$ lb-in$^2$.

The vertical and lateral flexural modes were estimated for the "simple-beam" suspension, implying that a single half-wave would be produced, with nodes at the ends of the beams.

Since the structure will be supported at four stations, equally spaced along its length, and the supports will be dynamically "soft" due to the very low frequency suspension, less than 2 Hz, it is probable that the structure/mass will be more likely to find a natural vibrating mode of the "free-free" or floating ship type, which is the frequency of a prismatic bar floating in space. Since there is no unbalanced force acting, the floating bar must follow the principle of conservation of momentum. That is the momentum of the parts moving in one direction must at all times equal the momentum of the parts moving in the opposite direction. This establishes the location of the nodes for such a mode. They are in the general vicinity of the quarter-points. The exact expression for frequency is:

$$f = \frac{C \times a}{l^2}, \quad \text{where for this case, } C = 3.58. \text{ Also, } a = \sqrt{\frac{EIg}{Ad}}$$

A is the cross-section area and, d is the density, so Ad= wt per unit length.

The resulting natural frequencies are higher than those for simple-beam mode, and it is interesting that they are the same as those for a bar clamped at both ends.

Vertical Flexural

$$f = \frac{3.58}{406^2}\sqrt{\frac{10^7 \text{ psi} \times 22{,}500 \text{ in}^4 \times 386 \text{ in/sec}^2}{(5000/406, \text{ or } 12.3 \text{ lb/in})}} = \boxed{58 \text{ Hz}}$$

Lateral Flexural

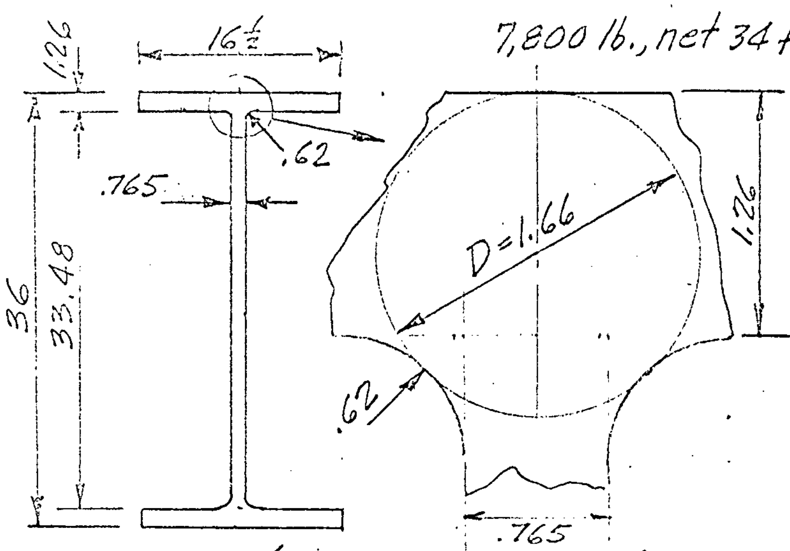
$$f = \frac{3.58}{406^2}\sqrt{\frac{10^7 \times 8{,}650 \times 386}{12.3}} = \boxed{36 \text{ Hz}}$$

Additional Data Relative to Technical Proposal - Appendix

Test Equipment Mount

## Investigate Structural Steel 36 WF 230 Section

7,800 lb., net 34 ft.




$I_x = 14{,}988\ in^4$

$I_y = 871\ in^4$

$J_p = 15{,}859\ in^4 \quad A = 67.73\ in^2$

$I_p = \frac{M}{A} J_p \ \frac{in^2\text{-}lb}{in}$

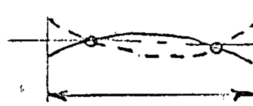
$M = 67.73 \times .284 = 19.2\ \frac{lb}{in}$

$I_p = .284 \times 15{,}859$

$I_p = 4500\ \frac{in^2\text{-}lb}{in}$

$E = 29 \times 10^6\ psi \ ; \ G = 11 \times 10^6\ psi$

## Lateral Flexure, "Free-Free" Mode



$L = 406\ in$

$$f_n = \frac{Ca}{L^2} \quad C = 3.58 \quad a = \sqrt{\frac{E I_y g}{A d}}$$

$$a = \sqrt{\frac{29 \times 10^6\ lb}{in^2} \times \frac{871\ in^4}{67.73\ in^2} \times \frac{386\ in}{sec^2} \times \frac{in^3}{.284\ lb}} = 7.1 \times 10^5\ \frac{in^2}{sec}$$

$$f_n = \frac{3.58 (\text{coeff. for first mode free-free}) \times 7.1 \times 10^5\ in^2/sec}{(406)^2\ in^2}$$

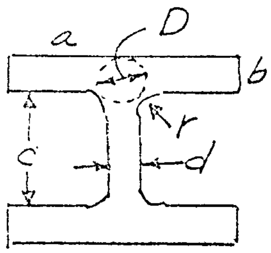
$$f_n = 15.4\ Hz \ \left(\frac{1}{sec}\right)$$

Frequency is estimated for beam alone. Loaded beam ($30\ lb/ft \times 34\ ft = 1{,}000\ lb$) would have slightly lower frequency

$$\text{say } \sqrt{\frac{7{,}800}{8{,}800}} \times 15.4 = \boxed{14.5\ Hz}$$

(Cont. 36WF230 section) Torsional Mode Vibration

The wide-flange shape will be "poor" in torsion (low frequency). Calculation must take into account the K-factor for non-linear stress (strain) distribution, as follows:




Total angle of twist $\theta = \frac{TL}{KG}$ where $T$ = torque, $L$ = length

and $K = 2K_1 + K_2 + 2\alpha D^4$ in.$^4$

$$K_1 = ab^3\left[\frac{1}{3} - 0.21\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right]$$

$$K_2 = \frac{1}{3}cd^3$$

$$\alpha = \frac{t}{t_1}\left(0.15 + 0.1\frac{r}{b}\right)$$

$t = b$ if $b < d$
$t = d$ if $d < b$
$t_1 = b$ if $b > d$
$t_1 = d$ if $d > b$

These expressions are based on the integration of elastic strain energy and the membrane analogy, and give results that are within 5% for normal shapes. (See Roark, Table IX, Case 17).

Unit torsional stiffness $\left(\frac{\text{Torque}}{\text{radian}}\right) = \frac{T}{\theta} = \frac{KG}{L}$

$a = 16.5 \quad b = 1.26 \quad c = 33.48 \quad d = .765 \quad r = .62 \quad D = 1.66$

$t = d \ (d < b) = .765 \quad t_1 = b \ (b > d) = 1.26$

$$\alpha = \frac{.765}{1.26}\left(0.15 + 0.1 \cdot \frac{.62}{1.26}\right) = .122 \quad D^4 = 7.6 \quad \alpha D^4 = .925$$

$$K_1 = (16.5)(1.26)^3\left[\frac{1}{3} - 0.21\frac{(1.26)}{16.5}\left(1 - \frac{(1.26)^4}{12(16.5)^4}\right)\right] = 10.5 \text{ in}^4$$

$$K_2 = \frac{(33.48)(.765)^3}{3} = 5.05 \text{ in}^4$$

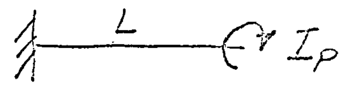
$$K = 2(10.5) + 5.05 + 2(.925) = \underline{27.9 \text{ in}^4}$$

(Cont. 36 WF 230) Torsional Mode

$k$, Unit Torsional Stiffness $\frac{T}{\theta} = \frac{KG}{L} = \frac{27.9\ in^4 (11 \times 10^6)\ lb/in^2}{406\ in}$

$$k = \frac{T}{\theta} = 755{,}000\ \text{in-lb/radian}$$

Clamp-Free (lowest fundamental)




$$f = \frac{1}{2\pi}\sqrt{\frac{k\,g}{I_p}} = \frac{1}{2\pi}\sqrt{\frac{.755 \times 10^6 \times 386}{4.5 \times 10^3 \times 406}} = 2\ H_z\ \left(\frac{1}{sec}\right)$$

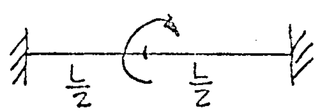
Again, this is for unloaded beam.

1,000 lb load @ 18-in radius adds $(1{,}000)(18)^2 = 324 \times 10^3$ $in^2$-lb

$$\frac{I'_p}{I_p} = \frac{(4.5)(406) + 324}{(4.5)(406)} = \frac{2154}{1830} = 1.18$$

$$\text{Loaded frequency} = \frac{2}{\sqrt{1.18}} = \boxed{1.8\ H_z}\ \left(\frac{1}{sec}\right)$$

Clamp-Clamp




$k' = \frac{2k}{\frac{1}{2}}$ (twisting 2 bms, $\frac{L}{2}$ long) $k' = 4k$

$$f' = f\sqrt{4} = 2f$$

$$f' = \underline{\underline{3.6\ Hz}}\ \left(\frac{1}{sec}\right)$$

Other modes would be possible at $\frac{5f}{2}$, $\frac{7f}{2}$, $2f'$, $3f'$, etc.
4.5 Hz
6.3 Hz
7.2 Hz
10.8 Hz etc.

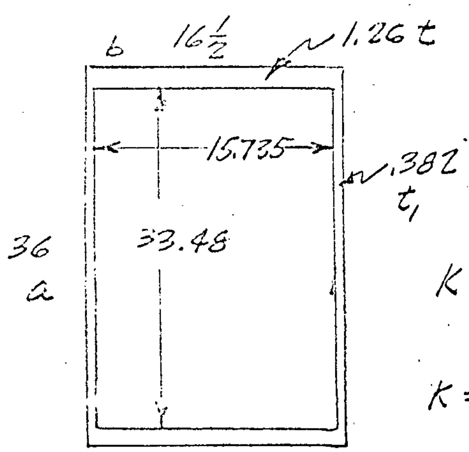
The low frequencies indicate the relatively weak torsional modes (approximately in frequency range of the isolator system, where transmission is > 1) into which much of the disturbing energy can cross-couple.

Additional Data Relative to Technical Proposal - Appendix

Test Equipment Mount

## Comparison of Flanged Beam (36 WF 230) with Box Structure Made of Identical Material

Simply for comparison, consider a box structure hypothetically made by using half the WF's web on each side:




$I_x$ = same $\quad I_y = \frac{(36)(16.5)^3 - (33.48)(15.735)^3}{12}$

$I_y = 2{,}585 \text{ in}^4$

$J_p = 14{,}938 + 2{,}585 = 17{,}573 \text{ in}^4$

$I_p = .284 \times 17{,}573 = 5{,}000 \ \frac{\text{in}^2\text{-lb}}{\text{in}}$

$$K = \frac{2tt_1(a-t)^2(b-t_1)^2}{at + bt_1 - t^2 - t_1^2}$$

$$K = \frac{(.962)(34.74)^2(16.12)^2}{45.3 + 6.3 - 1.59 - .12} = \underline{6{,}000 \text{ in}^4}$$

(Compared with $\underline{27.9}$)

Torsional Stiffness, $\tau = \frac{KG}{L} = \frac{6{,}000 \times 11 \times 10^6}{406} = 162 \times 10^6$ in-lb/radian

Clamp-Free $f = \frac{1}{2\pi}\sqrt{\frac{162 \times 10^6 \times 386}{5{,}000 \times 406}} = 29 \text{ Hz}$ (unloaded)

or $\underline{\underline{26 \text{ Hz}}}$ (loaded)

So we see that the box is 210 times as stiff and will resonate at a frequency $14\frac{1}{2}$ times higher than the WF beam shape.

It is also better in lateral free-free flexure:

$$f_{lat} = 14.5 \text{ Hz}\sqrt{\frac{2{,}585}{871}} = \underline{\underline{25 \text{ Hz}}}$$

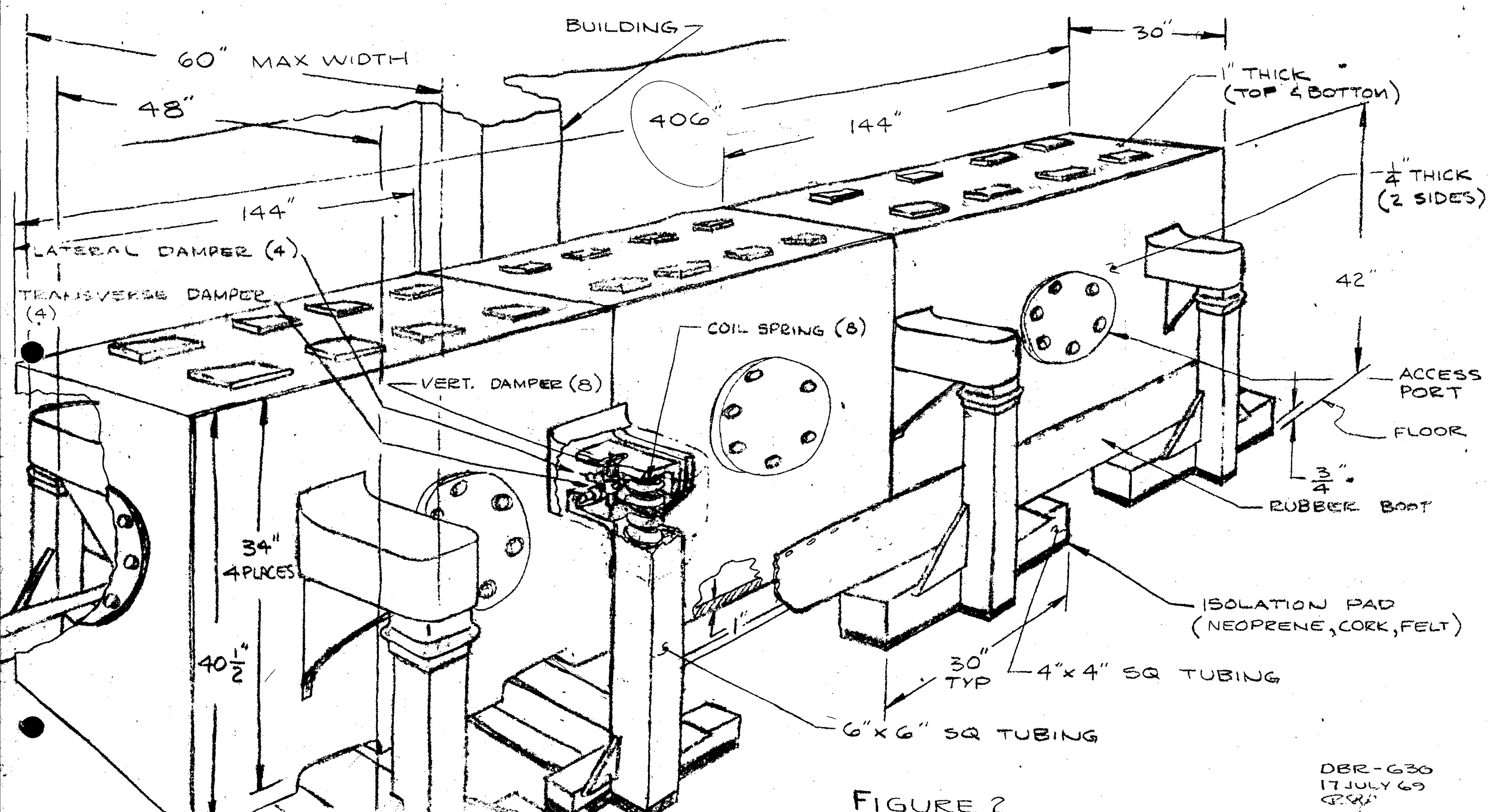
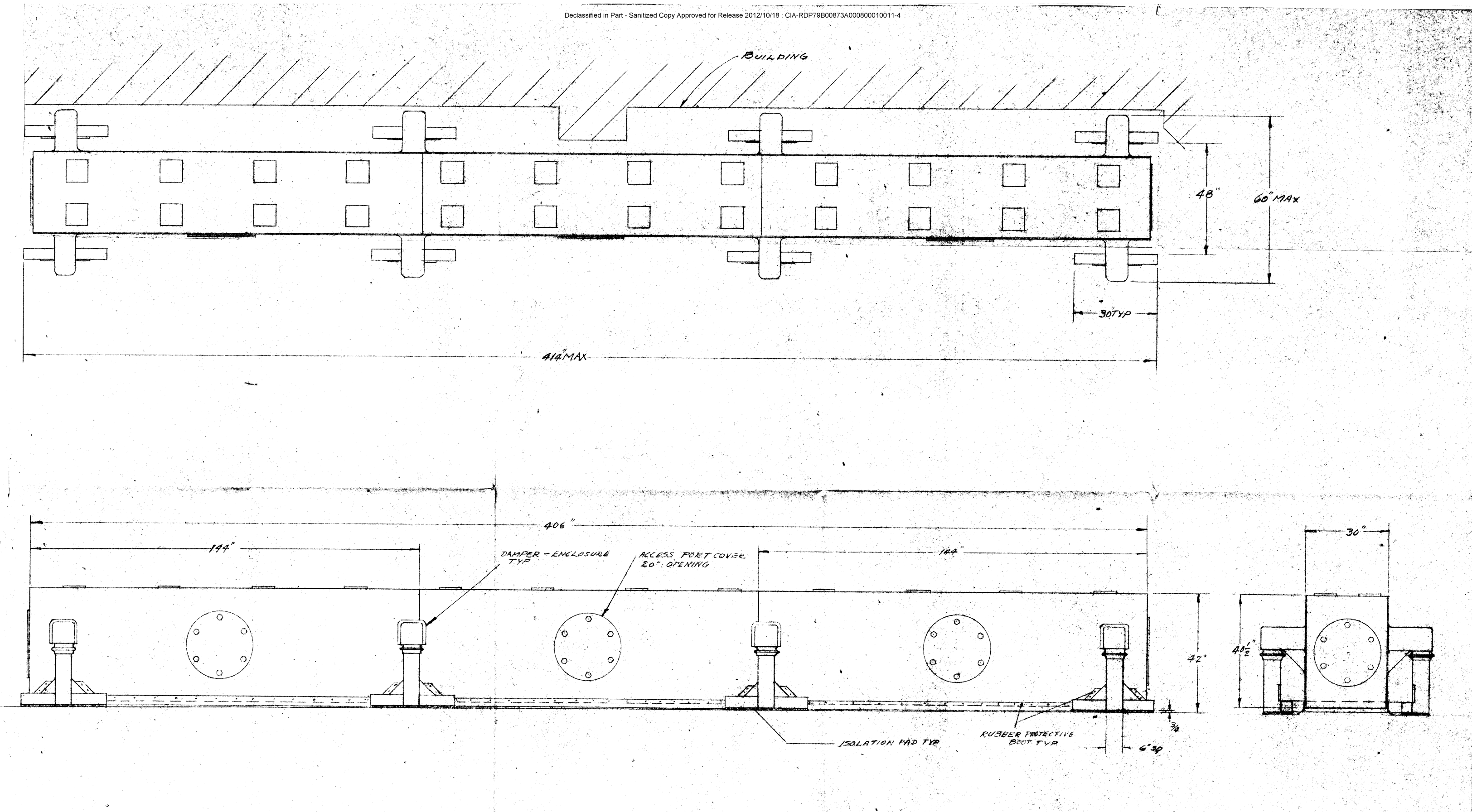



FIGURE 2

DBR-630
17 JULY 69




STAT

SCALE 3/4" = 1 ft
Drawn by [illegible] 7/24/59

TEST EQUIPMENT MOUNT
DBR 630
FIG 2A